INSTRUCTIONAL PLAN – Day 2

Subject: Trigonometry
Topic: Other Trigonometric Ratios, Relationships between Trigonometric Ratios, and Inverses
Target Learners: College Students
Objectives:

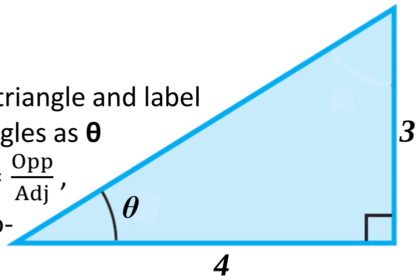
At the end of the lesson, students will be able to define the three other trigonometric ratios and derive all the five other ratios given just one ratio; and explain the meaning of an inverse and be able to use the inverses of sine, cosine and tangent in solving right triangles.

Instructional Activity	**Description of the Activity**
1. Motivation	Use the Powerpoint presentation (Time for a Walk.pptx) and the following as a hook: "From a parking lot you want to walk to a house on the ocean. The house is located 1500 feet down a paved path that parallels the beach, which is 500 feet wide. Along the path you can walk 300 feet per minute, but in the sand on the beach you can only walk 100 feet per minute. (a) What is the fastest time you can walk from the parking lot to the house? (b) For what angle θ would this fastest time be? (c) What would be the distance walked on the paved path? On the beach?" (Source: Sullivan and Sullivan, 2009) The presentation will show an illustration of the problem with proper labels. Say, "In order to answer the first two questions, we need to know how to find out the values of other trigonometric ratios, given just one of them, and we also need to understand the concept of inverses." **Technological requirements:** **Software – MS Powerpoint** **Hardware – computer, projector**
2. Objective	At the end of the lesson, students will be able to: • define cotangent, secant and cosecant. • find the values of other trigonometric ratios, given just one of them, by understanding the relationship between all six trigonometric ratios. • explain the concept of inverses and be able to use the inverses of sine, cosine and tangent in solving right triangles.
3. Prerequisite	Students will need prior knowledge about angles, triangles, ratio and proportion, exponents and radicals, solutions to algebraic equations, properties of right triangles, Pythagorean Theorem, and the three primary trigonometric ratios .
4. Information and examples	1. Define the three other trigonometric ratios. • $\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$ • $\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$ • $\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$ 2. Allow students to make the connection between sine and cosecant, cosine and secant, and tangent and

cotangent.

- $\csc\theta = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$

3. Show students the rational relationship of sine and cosine to tangent and cotangent, and let them derive the other rational relationships between the trigonometric ratios.
   - $\tan\theta = \frac{\sin\theta}{\cos\theta} = \sin\theta\sec\theta = \frac{\sec\theta}{\csc\theta}$
   - $\cot\theta = \frac{\cos\theta}{\sin\theta} = \cos\theta\csc\theta = \frac{\csc\theta}{\sec\theta}$
4. Sample problem:
   Given $\tan\theta = \frac{3}{4}$, find the values of the other five trigonometric ratios.
   Solution:
   - Draw a right triangle and label one of the angles as **θ**
   - Since $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$, the side opposite the angle would be 3 units and the side adjacent to the angle would be 4 units.



   - Use Pythagorean Theorem to get the length of the hypotenuse (5 units).
   - Since we now know the lengths of each side, we can use the definitions to find the values of the other trigonometric ratios:
     - $\sin\theta = \frac{3}{5}$
     - $\cos\theta = \frac{4}{5}$
     - $\cot\theta = \frac{4}{3}$
     - $\sec\theta = \frac{5}{4}$
     - $\csc\theta = \frac{5}{3}$
5. Call 2 to 3 pairs of students to read their homework regarding inverses. Ask the class for reactions regarding the pairs' definition and examples of inverses.
   Sample answer:
   An inverse is a reverse action of an operation. In essence, it undoes what the original operation did.
   For instance, to get from his home to his school, a student needs to walk 500 meters east and 200 meters north. To get from his school to his home, the student must then travel the same distance but in reverse order and on the opposite directions, that is, 200 meters south and then 500 meters west. This action brings him back to where he started, which is his home, thus undoing what has been done on his walk to school.
   (Source: McKeague & Turner, 2008)
6. Define the inverse of sine, cosine and tangent as the angle that would produce the ratios sine, cosine or tangent, respectively.

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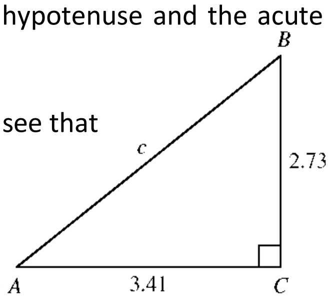
7. Sample problem: A right triangle has legs 2.73 and 3.41 units, respectively. Find the hypotenuse and the acute angles of the right triangle.
Solution:
From the illustration, we see that
$\tan A = \frac{2.73}{3.41}$
therefore, $A = \tan^{-1}\frac{2.73}{3.41}$
$= 38.7^o$
$B = 90^o - A = 90^o - 38.7^o$
$= 51.3^o$


By Pythagorean Theorem, $c = \sqrt{2.73^2 + 3.41^2} = 4.37$
Ask students if they could find other ways to solve the problem.
(Source: McKeague & Turner, 2008)
8. Build the students' vocabulary with the following terms:
• Inverse – an operation that undoes what the original operation did.
• Sine Inverse ($\sin^{-1}$) – the inverse operation of sine. It is an angle that would produce the sine ratio.
• Cosine Inverse ($\cos^{-1}$) – the inverse operation of cosine. It is an angle that would produce the cosine ratio.
• Tangent Inverse ($\tan^{-1}$) – the inverse operation of tangent. It is an angle that would produce the tangent ratio.
Technological requirement:
Hardware – scientific calculator
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<td>5. Practice and feedback</td>
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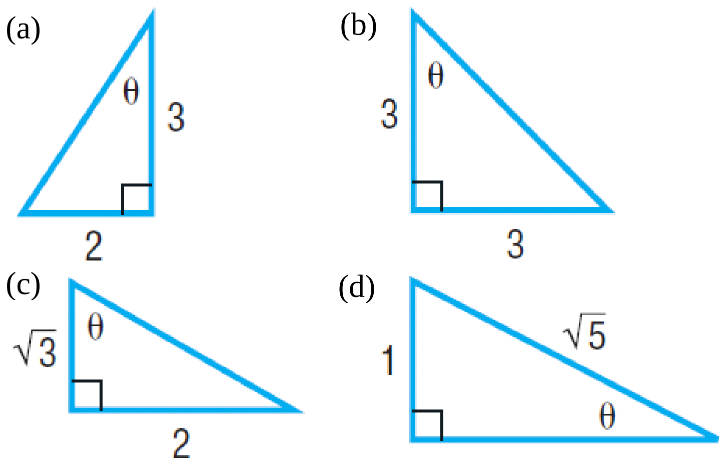
1. Allow the students to answer the following problems: Find the six trigonometric ratios of the angle θ in each figure. Find the measure of the angle θ as well.


Answers:
(a) $\sin\theta = \frac{2}{\sqrt{13}}$; $\cos\theta = \frac{3}{\sqrt{13}}$; $\tan\theta = \frac{2}{3}$
$\csc\theta = \frac{\sqrt{13}}{2}$; $\sec\theta = \frac{\sqrt{13}}{3}$; $\cot\theta = \frac{3}{2}$; $\theta = 33.7^o$
(b) $\sin\theta = \frac{1}{\sqrt{2}}$; $\cos\theta = \frac{1}{\sqrt{2}}$; $\tan\theta = 1$
$\csc\theta = \sqrt{2}$; $\sec\theta = \sqrt{2}$; $\cot\theta = 1$; $\theta = 45^o$
(c) $\sin\theta = \frac{2}{\sqrt{7}}$; $\cos\theta = \frac{\sqrt{3}}{\sqrt{7}}$; $\tan\theta = \frac{2}{\sqrt{3}}$
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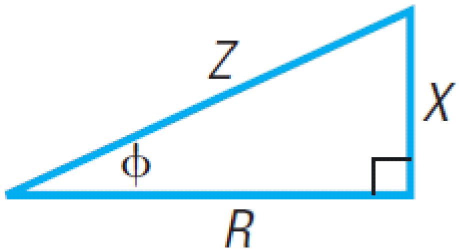
$\csc\theta = \frac{\sqrt{7}}{2}$; $\sec\theta = \frac{\sqrt{7}}{\sqrt{2}}$; $\cot\theta = \frac{\sqrt{3}}{2}$; $\theta = 49.1^o$

(d) $\sin\theta = \frac{1}{\sqrt{5}}$; $\cos\theta = \frac{2}{\sqrt{5}}$; $\tan\theta = \frac{1}{2}$

$\csc\theta = \sqrt{5}$; $\sec\theta = \frac{\sqrt{5}}{2}$; $\cot\theta = 2$; $\theta = 26.6^o$

2. A resistor and an inductor connected in a series network impede the flow of an alternating current. This impedance ***Z*** is determined by the reactance ***X*** of the inductor and the resistance ***R*** of the resistor. The three quantities, all measured in ohms, can be represented by the sides of a right triangle as illustrated.



From the Pythagorean Theorem, $Z^2 = X^2 + R^2$. The angle ϕ is called the phase angle. Suppose a series network has an inductive reactance of ***X*** = 400 ohms and a resistance of ***R*** = 600 ohms.

(a) Find the impedance Z. ($200\sqrt{13} \approx 721.1$ ohms)

(b) Find the phase angle ϕ. (33.7^o)

3. Ask the students to answer the question posted at the beginning of the lesson. Suggest that they use a graphing calculator for (a) and (b):

From a parking lot you want to walk to a house on the ocean. The house is located 1500 feet down a paved path that parallels the beach, which is 500 feet wide. Along the path you can walk 300 feet per minute, but in the sand on the beach you can only walk 100 feet per minute.

(a) What is the fastest time you can walk from the parking lot to the house? (9.7 min)

(b) For what angle θ would this fastest time be? (70.5^o)

(c) What would be the distance walked on the paved path? (1323 ft) On the beach? (530 ft)

(Source: Sullivan & Sullivan, 2009)

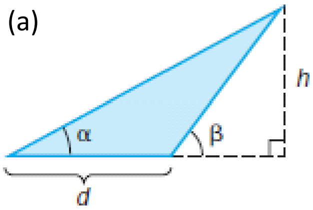
Technological requirements:

Hardware – scientific calculator, graphing calculator

6. Additional examples

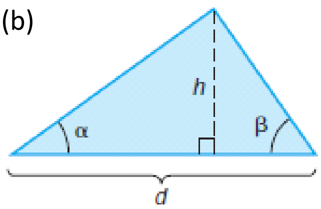
1. Express ***h*** in terms of ***d***, and the cotangents of ***α***, and **β** in the following figures:

(a)



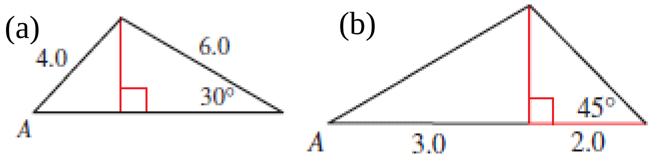
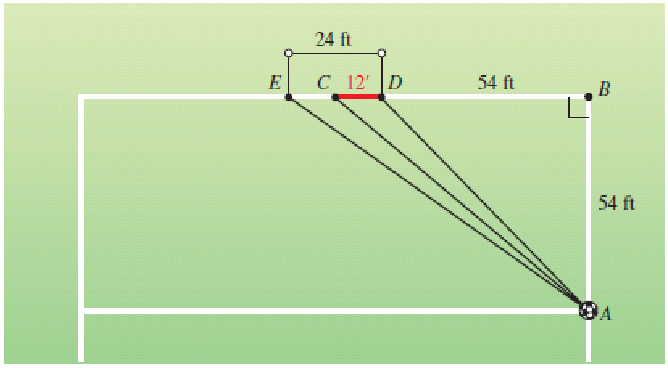
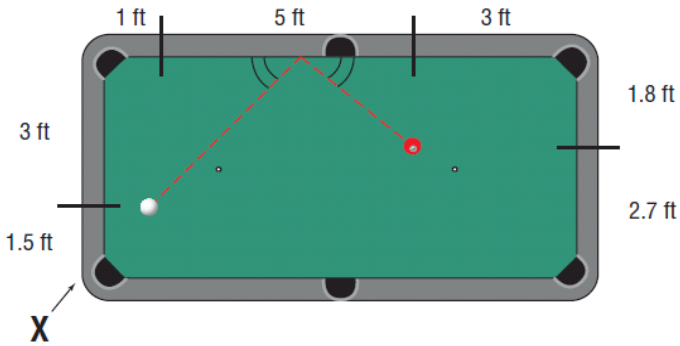
$\left(h = \frac{d}{\cot\alpha - \cot\beta}\right)$

(b)



$\left(h = \frac{d}{\cot\alpha + \cot\beta}\right)$

Source: Barnett, R. et al (2011)

	2. If a circle of radius 4 centimeters has a chord of length 3 centimeters, find the central angle that is opposite this chord (to the nearest degree). (44^o) Source: Barnett, R. et al (2011) 3. Find the measure of angle A to the nearest degree:  (49^o) (34^o) Source: McKeague & Turner (2008) **Technological requirement:** **Hardware – scientific calculator**
7. Additional practice and feedback	Short Pop Quiz: 1. **Penalty Kick.** A penalty kick is taken from a corner of the penalty area at position A (see figure below). The goalkeeper stands 6 feet from the goalpost nearest the shooter and can thus block a shot anywhere between the middle of the goal and the nearest goalpost (segment CD). To score, the shooter must kick the ball within the angle CAE. Find the measure of this angle to the nearest tenth of a degree.  Answer: 4.6^o Source: McKeague & Turner (2008) 2. **Calculating Pool Shots**. A pool player located at X wants to shoot the white ball off the top cushion and hit the red ball dead center. He knows from physics that the white ball will come off a cushion at the same angle as it hits a cushion. Where on the top cushion should he hit the white ball?  Answer: 4.125 ft from the upper-left corner Source: Sullivan & Sullivan (2009) **Technological requirement:** **Hardware – scientific calculator**

8. Summary	1. Cotangent, secant and cosecant are the three other trigonometric ratios. Cotangent is the reciprocal of tangent; secant is the reciprocal of cosine; and cosecant is the reciprocal of sine. 2. Tangent can also be expressed as the ratio of sine to cosine, or of secant to cosecant. Cotangent is likewise expressed as the ratio of cosine to sine, or of cosecant to secant 3. Inverses undo the operation done. Thus, the inverse of a trigonometric ratio is the angle at which the ratio between the specific sides was defined. 4. Knowledge of all the trigonometric ratios, their relation to each other, and the inverses of the trigonometric ratios can help solve problems involving right triangles and even those problems wherein right triangles are not apparent, such as sports, electricity, etc.
9. Homework	1. Separate the class into groups of 4 or 5 students. They will work on the Design A Game project, which will be submitted electronically at 10PM on the night before Day 5 and to be presented on Day 5 for this unit. 2. Journal/blog entry in their CMS pages about what they have learned from the session and their insights on how this new knowledge can be applied in their lives.

Evaluation/Assessment: Class participation, to be assessed using the Classwork/Participation Rubric (generated from iRubric – https://www.iRubric.com/), and short pop quiz worth 5 points each item.

References:

Aufmann, Richard N., Barker, Vernon C. and Nation, Richard D. (2011). *College Algebra and Trigonometry, 11th ed.*

Barnett, Raymond A. et al (2008). *College Algebra with Trigonometry, 9th ed.*

Larson, Ron (2012). *Algebra and Trigonometry: Real Mathematics, Real People, 6th ed.*

McKeague, Charles P. and Turner, Mark D. (2008). *Trigonometry, 7th ed*.

Sullivan, Michael and Sullivan, Michael III (2009). *Algebra & Trigonometry, 6th ed.*

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